Wavelet Packet Transform for Analysis of Time Series with Chaotic Structure

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The proposed method of optimal wavelet packet tree design identifies the characteristics of spectral structure for chaotic-like signals. This method has been introduced using the example of logistic mapping with numerous control parameters. This method can be used to collect information for decision support systems.

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Introduction

A lot of information, biological, physical, technological processes have a complex fractal structure. It showed many researches which were held in the past decades. Mathematical models of complex systems exhibiting irregular dynamics are both random and deterministic chaotic processes. The methods of data mining for modeling and forecasting of complex processes are used in recent years more often. Decision support system (DSS) for the study of the fractal structure of the time series was investigated in [1]. The unit defining the information about complexity of the system was added in the knowledge base for more qualitative researches. Researches of different fractal time series using this modified DSS were investigated. They showed the ability to recognize different states of the system.

A wavelet analysis is one of the powerful tools for research and classification of time series. In particular multiresolution analysis it allows to decompose time series into components with different frequency ranges. Using the wavelet characteristics as knowledge for DSS allows to recognize the characteristic features of fractal signals.

Packet wavelet transform is type of wavelet transforms, which is widely used to compress and denoise the signals. It allows more accurately to adapt to features of the signals by optimal shape of the decomposition tree [2]. In this work the optimal wavelet decomposition tree is suggested to apply for recognition of chaotic signals in different chaos modes. Such approach allows to identify changes of frequency ranges that corresponds to the chaotic modes changes.

The purpose of work is comparative analysis of chaotic time series which was carried out using the package wavelet transform and can be used as knowledge in decision support system.

Decomposition of Time Series Using Wavelet Transform

Wavelet transformation of one-dimensional signal is its presentation as a generalized series or integral system of basis functions

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right),$$

which are obtained from the mother wavelet $\psi(t)$. Mother wavelet has properties of shift in time b and temporary scale a. Discrete wavelet transform (DWT) is constructed using multiresolution analysis. The main idea is representation of the signal in the form of the set of its successive approximations.

Multiresolution analysis is to dividing of studied signal $X(t)$ into two components: approximating and detailing. After that the approximating component is divided to the N level of signal which has been set. As a result of decomposition the signal $X(t)$ is presented as a sum of approximating $approx_N(t)$ and detailing $detail_j(t)$ components:

$$X(t) = approx_N(t) + \sum_{k=1}^{N} detail_j(t) = \sum_{k=1}^{N_a} apr(N,k)\varphi_{N,k}(t) + \sum_{j=1}^{N}\sum_{k=1}^{N_j} det(j,k)\psi_{jk}(t),$$

where N is maximum level of decomposition, $apr(N,k) = \int\limits_{-\infty}^{\infty} X(t)\varphi_{Nk}(t)dt$ is approximating wavelet coefficients of level N, $det(j,k) = \int\limits_{-\infty}^{\infty} X(t)\psi_{jk}(t)dt$ is detailing wavelet coefficients of level j, N_j is number of the detailing coefficients of level N, N_a is number of the approximating coefficients of level N [2].

Selecting the type of wavelet function and the number of decomposition levels are an important issue for DWT. Typically the wavelet function is selected according to the time and frequency characteristics of each analyzed signal. The maximum level of decomposition depends on which frequency ranges are necessary to investigate.

The idea of wavelet packet analysis is to divide detailing components using the same method of decomposition. So packet DWT promotes better frequency localization. The result is a tree of decomposition, an example of it is shown on Figure 1 (right one).

Each node of the tree of packet DWT contains a set of wavelet coefficients that correspond to certain frequency range. Time series can be restored over packet wavelet coefficients that are in the terminal nodes of the tree.

Optimal Wavelet Tree

Packet wavelet transformation often graphically represent as a tree the root is the original signal. Packages are branches can be correlated with a certain frequency range. Packages that are not containing information about the signal can

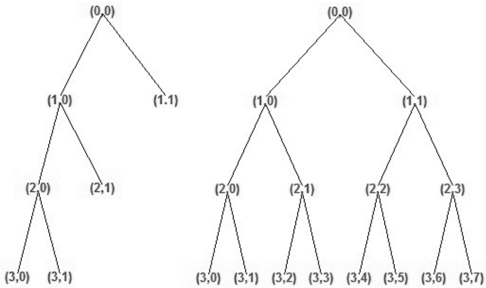

Figure 1. Tree of DWT (left) and packet DWT (right)

be considered as noise. Index of entropy is a measure of informativeness of a set of coefficients. The next types of entropy calculation are the most used in signal processing: Shannon entropy

$$E(s) = -\sum {s_i}^2 \log \left({s_i}^2\right);$$

norm of the space

$$E(s) = -\sum |s_i|^p, \ p \geq 1,$$

logarithmic energy

$$E(s) = -\sum \log({s_i}^2),$$

threshold entropy $E(s) = \sum i$, with $|s_i| > \varepsilon$, ε is a threshold value [3]. In all cases, the value s is an array of wavelet coefficients of the wavelet tree node.

The best tree is constructed on such scheme where node N is divided into two nodes N_1 and N_2 only if the sum of nodes entropy N_1 and N_2 smaller than the entropy of the node N. More distant nodes from the root of the tree contain a low-frequency coefficients and more information about researched signal. One can choose the best method for constructing wavelet tree for a specific task by changing the function of node entropy and type of mother wavelet.

Construction of Wavelet Trees of Model Chaotic Signals

The main feature of chaotic systems is sensitive dependence to arbitrarily small changes in initial conditions. Iterated maps $x_{n+1} = f(C, x_n)$ where C is control parameter, are the most simple and intuitive mathematical chaotic models. Realizations of chaotic maps have complex fractal structure [4]. Logistic map is the most famous example of chaotic maps. This one-dimensional quadratic map defined as:

$$x_{n+1} = Ax_n(1 - x_n),$$

where A is is control parameter, $A\epsilon(0, 4], x_n\epsilon[0, 1]$.

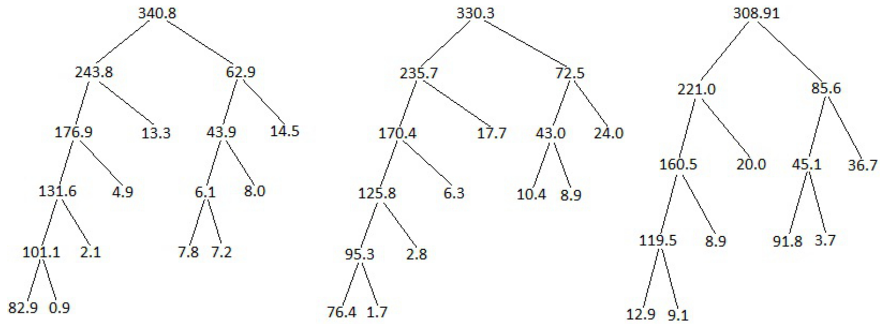

Figure 2. Optimal wavelet trees for logistic map with the values of the nodes entropy

Figure 2 shows the optimum wavelet trees for logistic maps with different chaotic modes $A = 3.7, 3.8, 3.9$. The corresponding values of the Lyapunov exponent is $\lambda = 0.19, 0.28, 0.48$.

Trees show the main frequency components of signals. They differ in structure and values of the nodes entropy. Correctly choosing mother wavelet and wavelet entropy one can uniquely identify the type a researched signal. In this work norm of the space entropy with parameter $p = 1$ and Daubechies order 4 wavelet was used.

Construction of Wavelet Trees for EEG Signals

EEG signal can be carried out using the methods which developed in the theory of dynamical chaos under the assumptions that the brain (or its part) is considered as a non-linear dynamic system is sensitive to initial conditions and EEG is the trajectory of this dynamic system in the phase space. So, it is assumed that the electrical activity described implicit chaotic system and despite the absence of a system of equations that model the bioelectric activity of the brain, it is possible to study the behavior of the system by its output data [5].

In the work we investigated EEG records of laboratory animals, which were divided into phases of wakefulness (AWAKE), slow-wave sleep (SWS) and rapid eye movement sleep (REM). Figure 3 shows the optimum wavelet trees for these EEG signals.

It is obvious that the optimal wavelet trees for EEG signals have a different structure and values of the nodes entropy. It allows identify the EEG signals. However, the optimal wavelet tree for EEG signals of REM sleep can consist of two or three levels. This is due to fact that it is phase of sleep is characterized by increased activity in the brain. Therefore, for more accurate analysis of such signals should also be used other information characteristics.

Conclusion

An optimal wavelet packet tree design makes it possible to establish the difference between signals, which have different chaotic modes. In common practice this

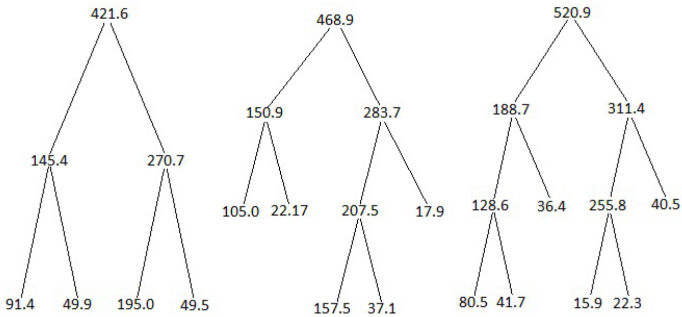

Figure 3. The optimum trees for EEG signals with the values of the nodes entropy

can be used to distinguish bioelectric signals. In common practice this can be used to distinguish bioelectric signals. It allows to research more correct mathematical models of time series that have fractal properties.

Further research is to develop the method that allows one to automatically classify time series from the shape of optimal wavelet decomposition tree and the values entropy in knots.

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